

AD-A127 232

RADIATION PATTERNS OF A DUAL REFLECTOR SYSTEM BASED ON
THE GEOMETRICAL OPTICS APPROXIMATION(U) NAVAL RESEARCH
LAB WASHINGTON DC J K HSIA(03 MAY 83 NRL-MR-5843

1/1

UNCLASSIFIED

F/G 9/5

NL

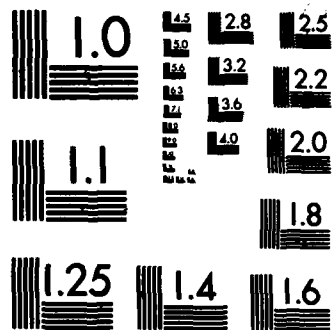
END

FILED

31

8-10

1010



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Memorandum Report 5049	2. GOVT ACCESSION NO. AD-A127232	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RADIATION PATTERNS OF A DUAL REFLECTOR SYSTEM BASED ON THE GEOMETRICAL OPTICS APPROXIMATION		5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL problem.
7. AUTHOR(s) J.K. Hsiao		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, DC 20375		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61153N; RR021-05-43; 53-1783-0-3
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE May 3, 1983
		13. NUMBER OF PAGES 26
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Reflector antenna pattern Antenna pattern		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this report the two-dimensional radiation patterns of an array-fed dual reflector system are computed and presented. For the purpose of preliminary study of the feasibility of this system, the pattern is computed by means of an approximate geometrical-option method. Results show that the radiation patterns of this system are well behaved, exhibiting no difficulty in beam steering. They also show that a modest sidelobe level can be achieved (30 dB or better). For better pattern synthesis and optimum array spacing, however, further work is required.		

DD FORM 1473

1 JAN 73

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 3102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

CONTENTS

INTRODUCTION	1
PATTERN COMPUTATION BY USE OF GEOMETRICAL-OPTICS METHOD	1
COMPUTED RESULTS	2
SIDELOBE DESIGN	3
CONCLUSIONS	4
REFERENCE	4
APPENDIX A	20
APPENDIX B	21

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By _____	
Distribution/ _____	
Availability Codes	
Dist	Avail and/or Special
A	



RADIATION PATTERNS OF A DUAL REFLECTOR SYSTEM BASED ON THE GEOMETRICAL OPTICS APPROXIMATION

1. INTRODUCTION

In a previous report [1], an electronically steerable dual reflector system fed by a phased array was discussed. The configuration of such a system is shown in Figure 1*. The cross section of the main reflector of this system is a parabola while the subreflector is an ellipse. The novelty of this system is that the two foci of the ellipse are located respectively at the center of the main reflector and the center of the feed array. Therefore, rays from all directions reflected at the center of the main reflector will converge at the center of the feed array. If the scan angle is small and the sizes and locations of both reflectors are properly chosen, rays within the required scan range will be distributed on the array face. The required feed array and subreflector sizes are minimized and the spillover loss can be very small.

In reference [1], a method for designing such a system is discussed, which is based on the geometrical-optics method. The same approach will be used in this report to compute a two-dimensional radiation pattern of such a system. It is known that the geometrical optics approach is only an approximation, particularly when interference effects are sought. To accurately compute the radiation patterns of such a system, a current distribution integral method must be used. However, such an approach is tedious and requires a major effort. At the present time, we are only interested in a preliminary study to determine the feasibility of such a system. Therefore the geometrical-optics method is used. If the results are promising, effort will be applied to the current distribution-integral method.

In this approach, a receiving case is assumed. All incoming waves are assumed to be planar with various incidence angles with respect to the main reflector. At each incidence angle a large number of rays is assumed and they are uniformly distributed on the main reflector. The path length of each ray from the incident wave front to the feed array is then computed, and the number of rays intersecting each element in the feed array is determined. Thus, the amplitudes and phases of these incoming waves on the feed array can be determined, and the radiation pattern can then be computed.

In this report, we wish to determine whether it is feasible to steer the beam off the broadside and how this will affect the sidelobe level. Secondly, we would like to determine whether a conventional method can be used for pattern synthesis. Third, we wish to know how closely the feed elements in the feed array must be spaced and how they can be phased to achieve beam steering.

2. PATTERN COMPUTATION BY USE OF GEOMETRICAL-OPTICS METHOD

Figure 1 shows a double reflector system where the surface R is the main reflector and S is the subreflector. The feed array is labeled A. Assume that a plane wave propagates toward the main reflector R with an incidence angle θ . To compute the radiation intensity at each incidence angle with respect to the main reflector, it is assumed that a large number of parallel rays are incident on the main reflector and that they are uniformly distributed in a

*This system was suggested by J.P. Shelton

Manuscript approved January 24, 1983.

plane perpendicular to their direction. These rays are reflected in accordance with Snell's law by both the main reflector and the subreflector, and they finally intersect the feed array. To compute the radiation pattern the phase and amplitude of the incident wave at the feed array must be known. At each incidence angle, the ray phase is determined by the distance traveled by the ray from the plane wave front to the main reflector and the doubly reflected path from the main reflector to the array surface. The ray path length from the wave front to a point on the main reflector (ℓ_0) is computed by using equation (A3) given in Appendix A. The computations of the reflected ray path lengths from the main reflector to the subreflector (ℓ_1) and from subreflector to the feed array (ℓ_2) are given in Appendix B. The phase of each ray at the array face is then modulo 2π of $(2/\lambda)(\ell_0 + \ell_1 + \ell_2)$ where λ is the wavelength of the incident wave. Since both main reflector and subreflector are curved surfaces, the rays that are uniformly incident on the main reflector are not necessarily uniformly incident on the feed array face. The number of rays, or flux incident per unit area at a point, represents the intensity of the incident field at that point. If a large number of rays (or flux) is assumed incident on the main reflector, and the feed array is divided into n sections (or n elements) where n is a much smaller number than that of the total number of rays, then the number of rays incident on each of the n sections represents the intensity of the incident field on that element. For pattern synthesis, a weighting function is usually applied to each element to achieve the desired low sidelobe level. For the present case, this weighting function must take into account this non-uniformity property. A critical design problem relates to the variation of incident field intensity on each element as a function of scan angle. If it changes too much at different scan angles, it may be impossible to achieve a desired pattern with a constant array weighting function. However, the examples which we have studied indicate that the field intensity on the array face is uniformly distributed and its pattern resembles that of a conventional linear array. Of course, in arriving at this conclusion, it is assumed that the array element is not a point source, has omnidirectional element pattern, and only receives energy from the direction of the wave front, or in the ray tracing region.

To steer the beam to an angle θ_0 , the required phase setting at each element can be achieved by averaging the path lengths of all rays which intersect that element and which are incident at an angle θ_0 to the main reflector. The required element phase setting is just the conjugate of this average phase. One may see that if the path phase of each individual ray within each element deviates too much from the average phase value, then it may be difficult to form a beam in the θ_0 direction. This can happen when the array element spacing is too large.

3. COMPUTED RESULTS

As an example, the radiation pattern of a double reflector system is computed and some of its results are presented here. The system computed here is designed by the method discussed in reference [1]. The dimensions of this system are as follows:

Main Reflector	50λ
Sub Reflector	20λ

Feed Array

16λ

Focal Length (Main Reflector) 84λ

Some computed patterns for scan angles at -2 degrees, 0 degree and 8 degrees are shown respectively in Figures (2a), (2b) and (2c). The feed array is uniformly excited and has an element spacing of $\lambda/2$. The patterns are plotted in a range from -20 degrees to 20 degrees which is the required field range of the system contemplated.

Similar patterns are plotted for the same system in Figures (3a), (3b), and (3c) at similar scan angles. However, in this case, the feed array is excited by a 30 dB Chebyshev weighting function.

Figures (4a) and (4b) show the radiation pattern of the same system except that the element spacing is $.9\lambda$ instead of $.5\lambda$.

From these figures the following conclusions are drawn:

a. When the feed array is excited uniformly, its radiation pattern is similar to that of a conventional uniformly weighted linear array. No appreciable degradation is evident when the beam is steered over a range from -2 degrees to 8 degrees.

b. A conventional weighting function such as the Chebyshev designed for 30 dB sidelobe yields similar results to those obtained with a conventional linear array. However, when attempts were made to further reduce sidelobes, they do not seem to work. This will be discussed in detail in the next section.

c. When element spacing increases beyond a certain limit, a kind of grating lobe appears. However, the exact point of limitation is not clear at this point. It varies from system to system. Further study in this area is required.

4. SIDELOBE DESIGN

Since the achievement of low sidelobes is very important, attempts were made to achieve lower sidelobe levels than in the examples shown in section 3. The first attempt is to use a conventional array approach in which 40 dB, 50 dB and 60 dB Chebyshev array weights are used. Patterns for such illumination weights are shown in Figures (5a), (5b) and (5c), respectively. The configuration of the system of these plots is the same as that of Figures 2 and 3. One may see from these figures that the attainable sidelobe level seems to be limited. Another approach to tapering the array illumination was also tried, as shown in Figure 6. This array illumination function has a cosine squared distribution with edge element 20 dB down from the center element. Again, the achievable sidelobe level is quite limited. Patterns at other scan angles for both illumination cases are similar; hence they are not shown.

In order to investigate why a conventional pattern synthesis method does

not work in this system, the phase and amplitude distribution across the array surface for plane waves incident at 0 and -2 degrees to the main reflector are plotted in Figures (7a) and (7b). Figure (7a) shows the amplitude distribution (or intensity of incident waves). Although it varies, it is reasonably uniform. Compensation of this variation with array illumination function does not seem to yield the desired result. Amplitude distributions of plane waves incident at other angles show similar properties.

Figure (7b) shows the phase distribution for the cases of 0 and -2 degrees. (Actual Phase, not mod. 2π) This phase distribution at each element is actually the average of the phases of all the incident waves which intersect with that array element. One may see that the variation of the phase across the array face has a quadratic distribution. A conventional linear array has a linearly progressive phase distribution and its pattern synthesis is based on this property. It is evident that, for an array of this type, a conventional linear array pattern synthesis method cannot produce the desired result. Further work along this line is required.

5. CONCLUSIONS

The results presented in this report are encouraging. The radiation pattern behave reasonably well. There seems to be no difficulty in beam steering and it also responds well to the conventional pattern control weighting function to a certain degree. However, there are certain problems which require further investigation, such as array element spacing and pattern synthesis. It has been pointed out that since these results are based on the geometrical-optics method, it is only an approximation. Because of these encouraging results, we recommend that further effort be extended. We recommend that this problem be rigorously formulated by use of the current-distribution integration method, and we also recommend that the three dimensional case be analyzed.

6. REFERENCE

- [1] "Ray Constraints on the Design of an Offset Feed Array for a Dual Reflector System Using a Guided Computer Search Method", J.K. Hsiao, NRL Memorandum Report 4742, 2 March 1982.



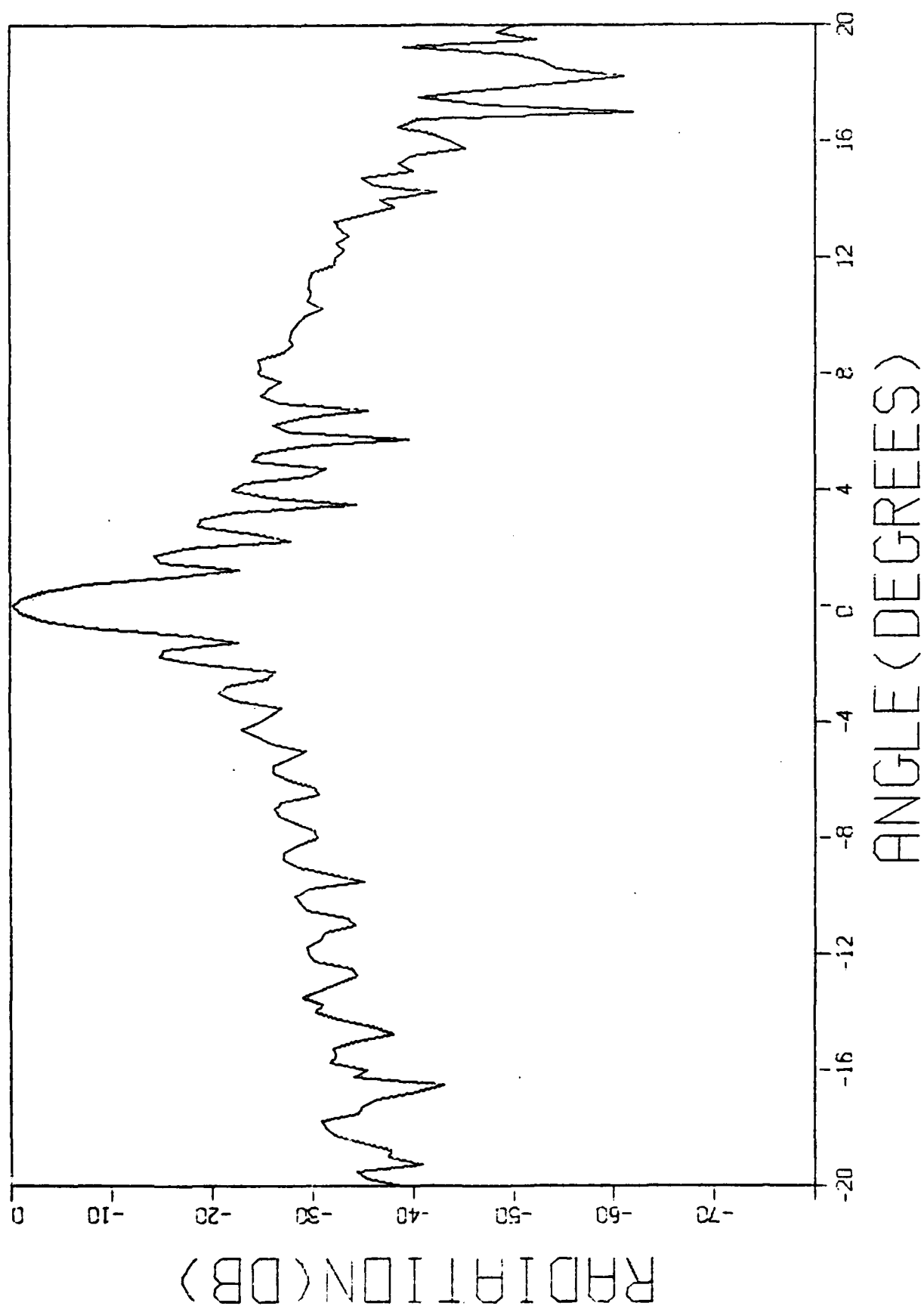


Fig. 2(a) — Radiation pattern, steering angle = 0, element spacing = $\lambda/2$.

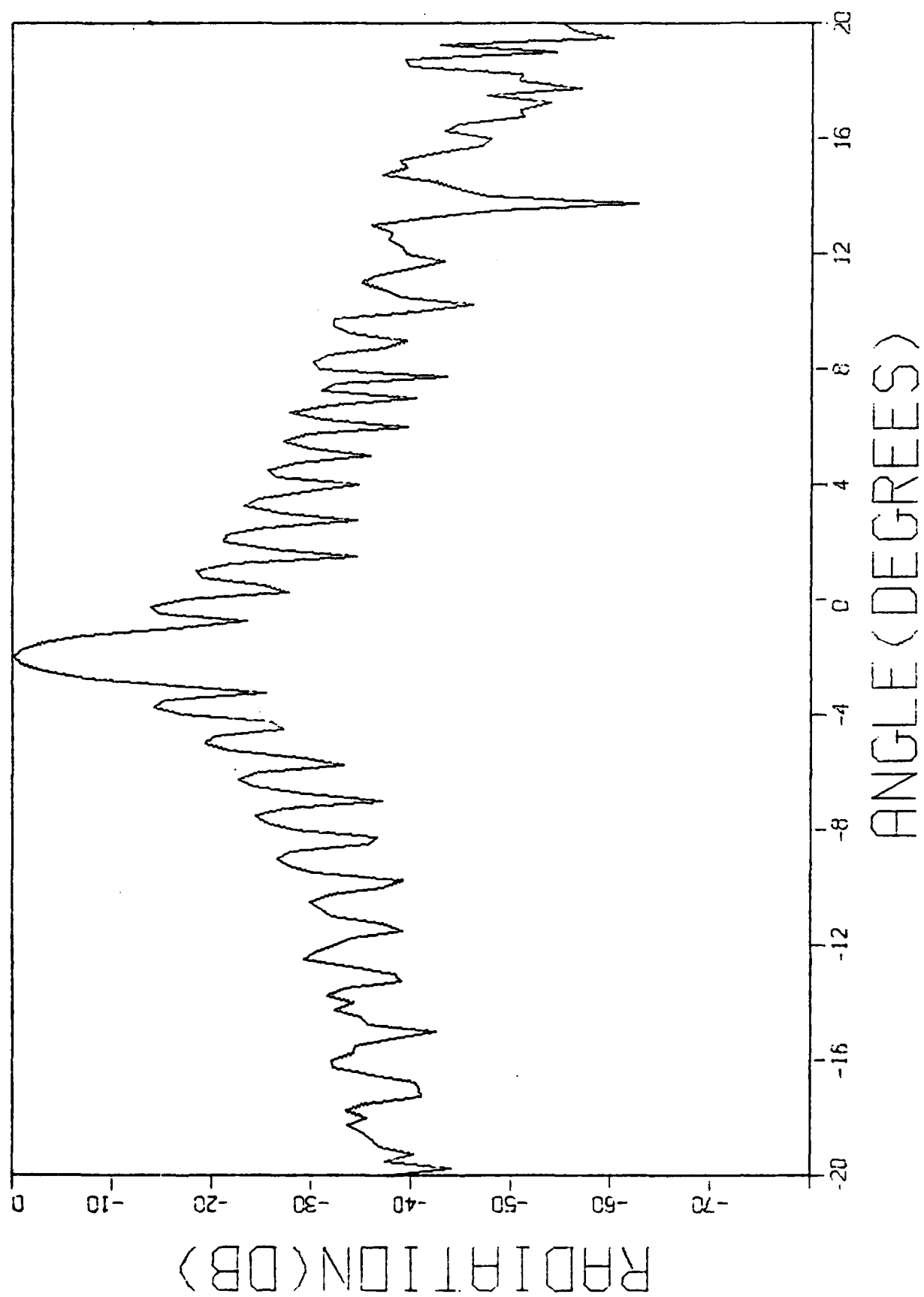


Fig. 2(b) — Radiation pattern, steering angle = -2° , element spacing = $\lambda/2$.

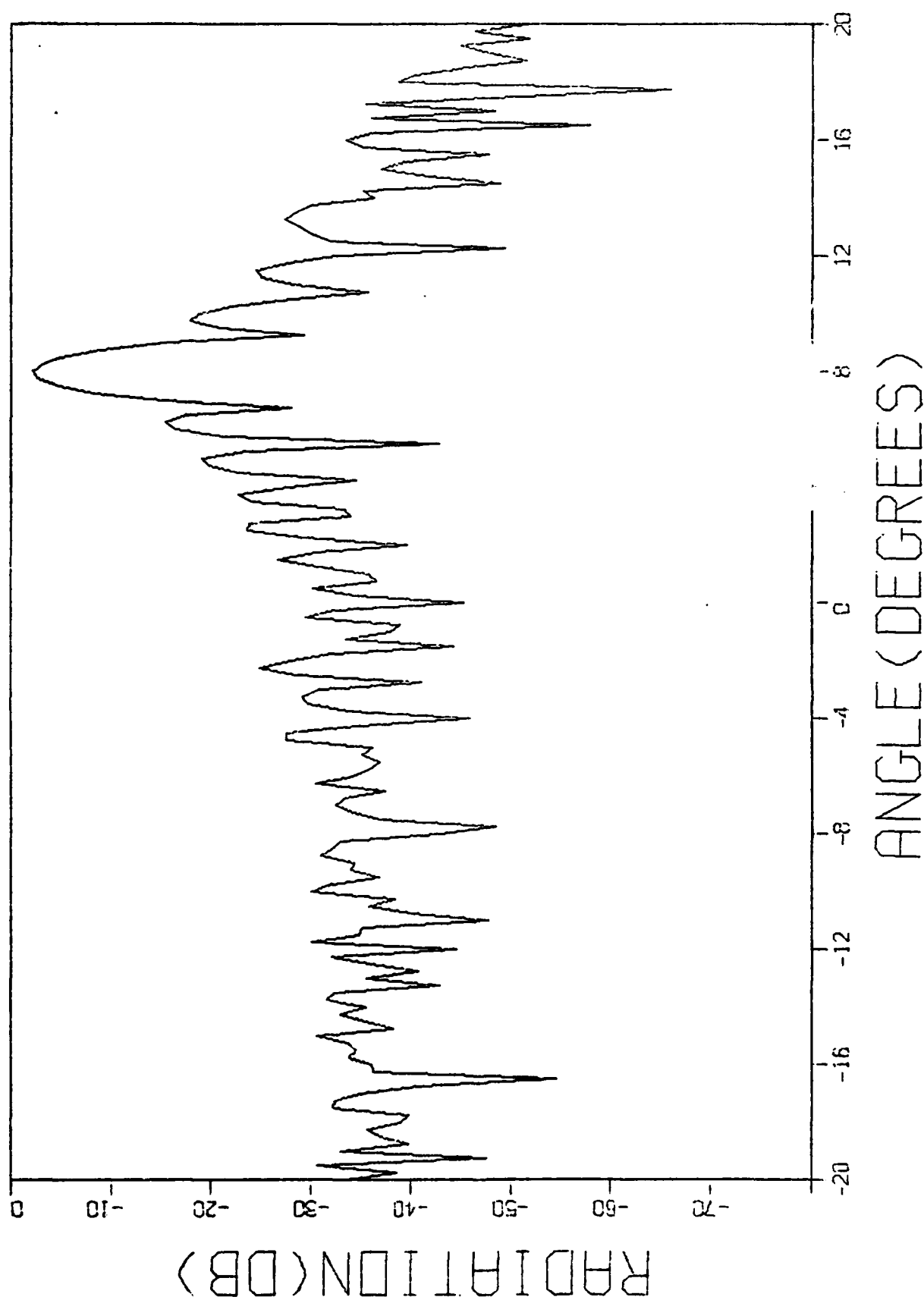


Fig. 2(c) — Radiation pattern, steering angle = 8° , element spacing = $\lambda/2$.

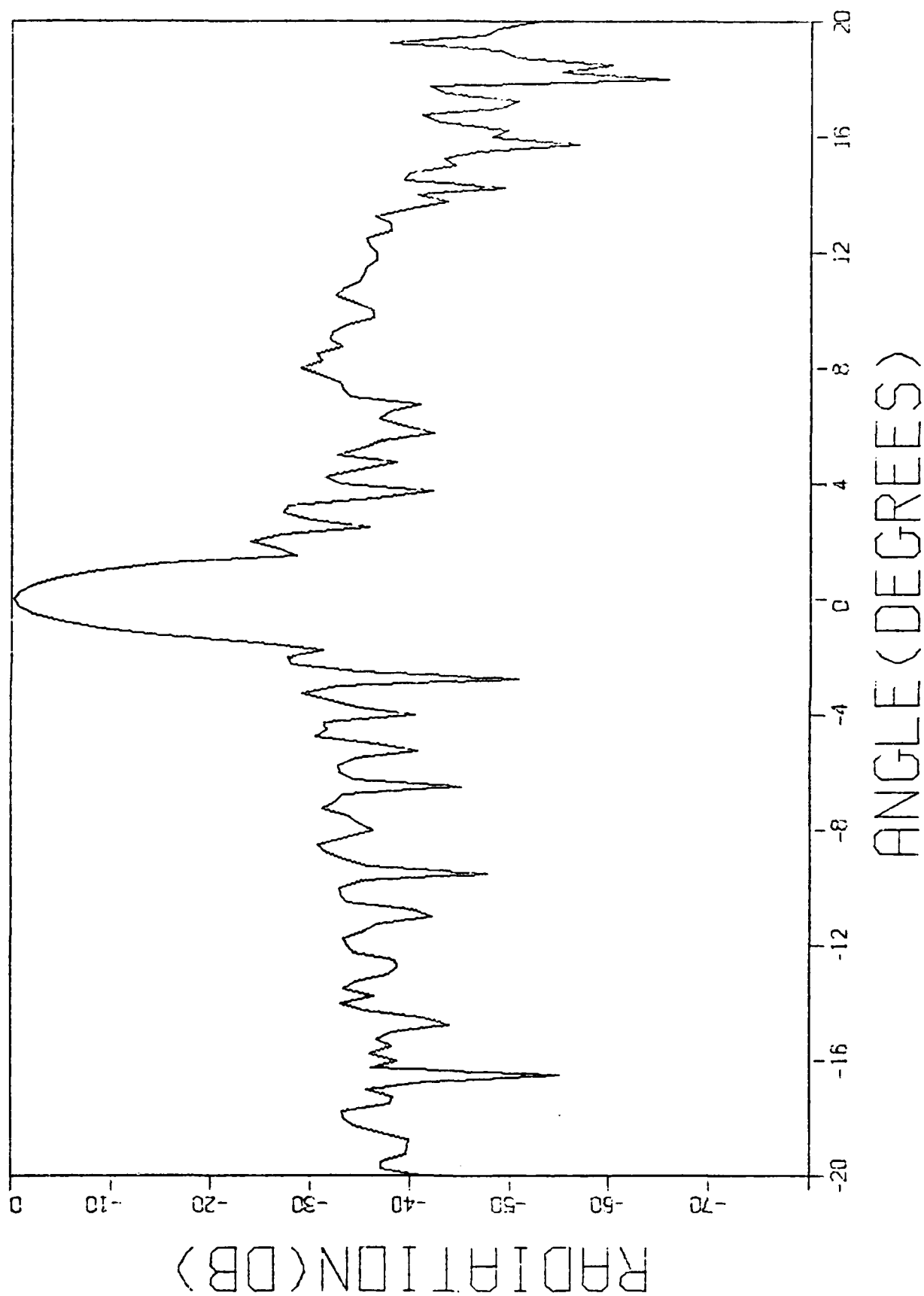


Fig. 3(a) — Radiation pattern, steering angle = 0° , element spacing = $\lambda/2$, 30 dB, Chebyshev weighting.

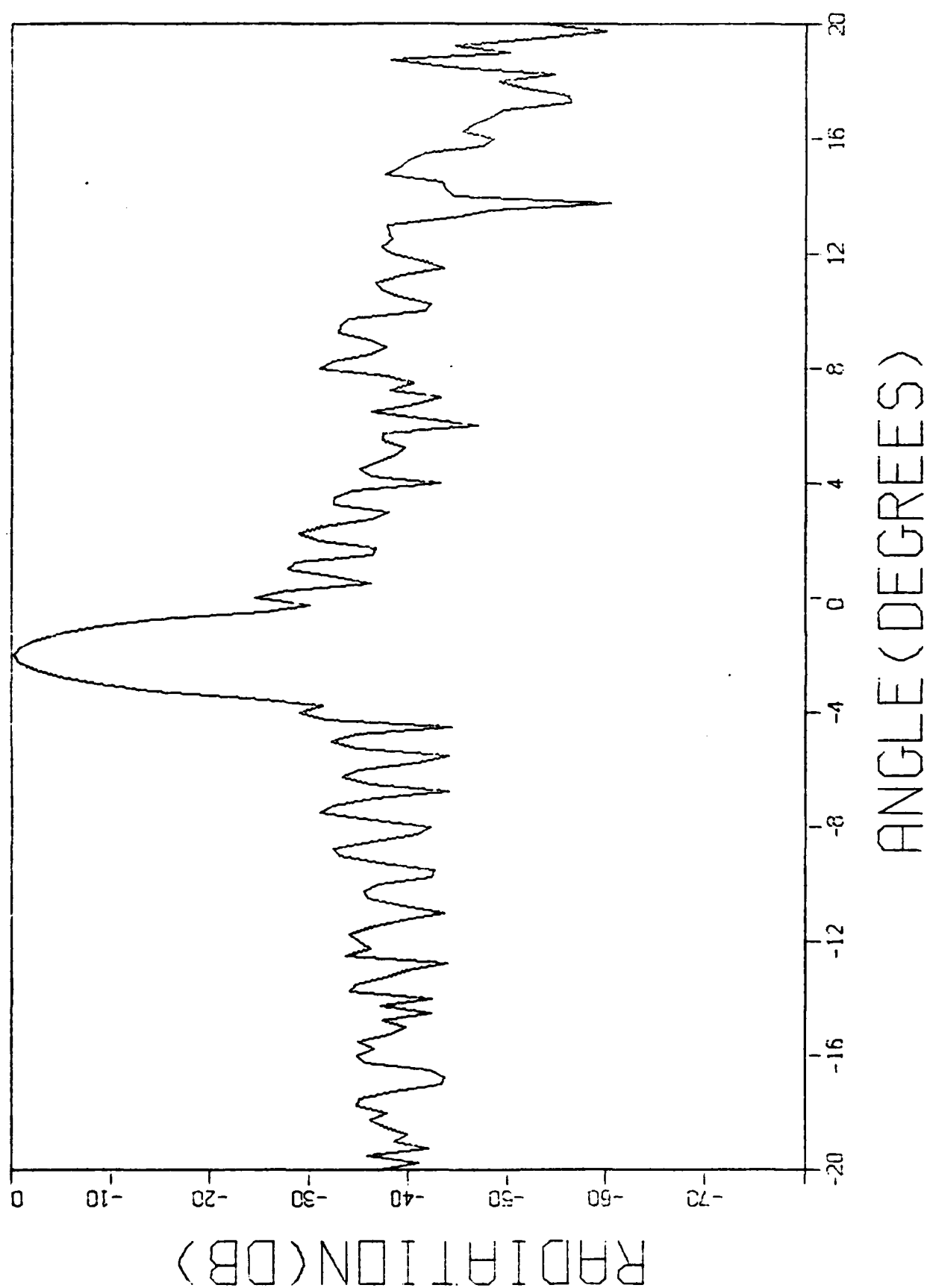


Fig. 3(b) — Radiation pattern, steering angle = -2° , element spacing = $1/2\lambda$, 30 dB Chebyshev weighting.

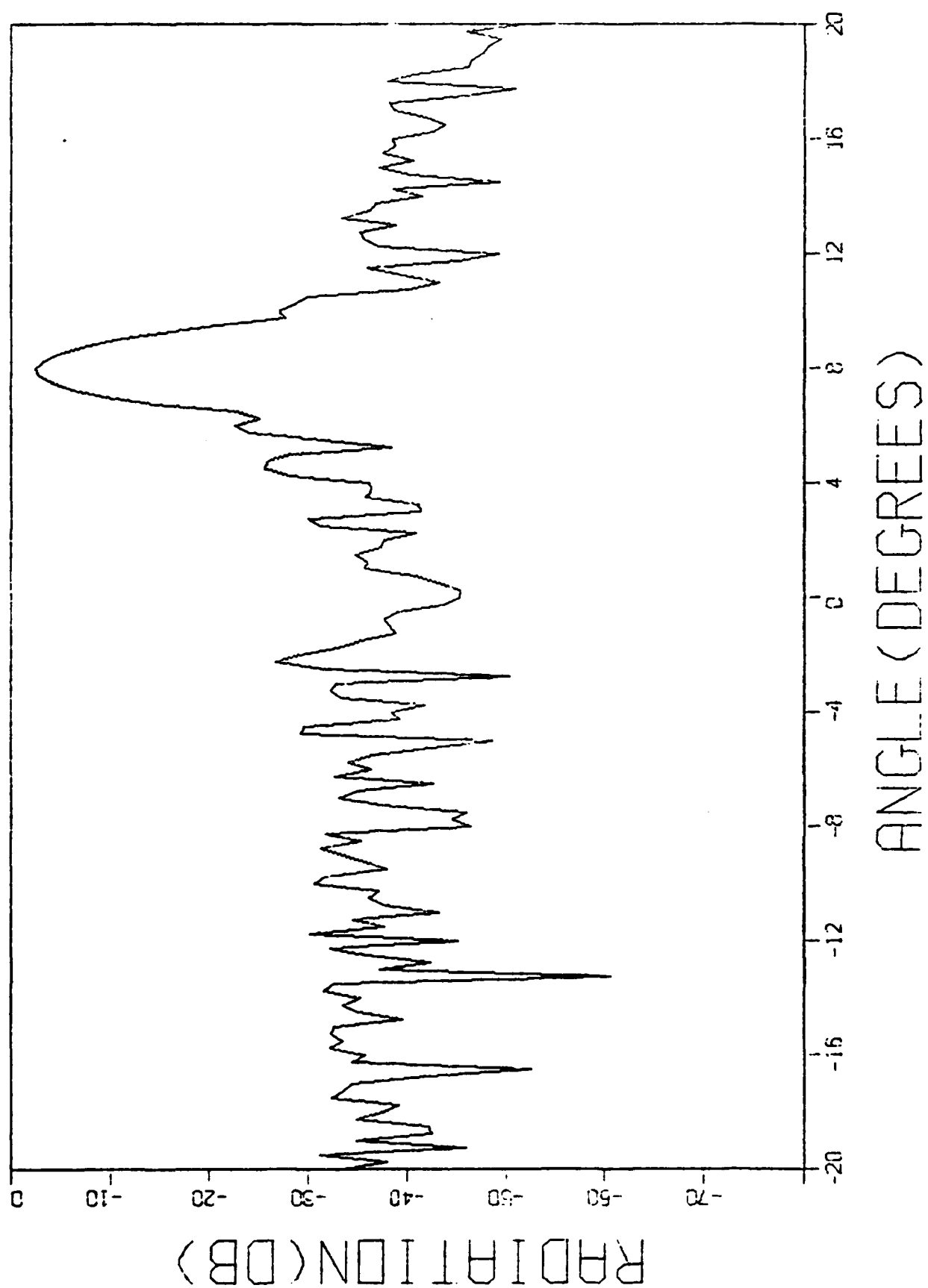


Fig. 3(c) — Radiation pattern, steering angle = 8° , element spacing = $\lambda/2$, 30 dB Chebyshev weighting.

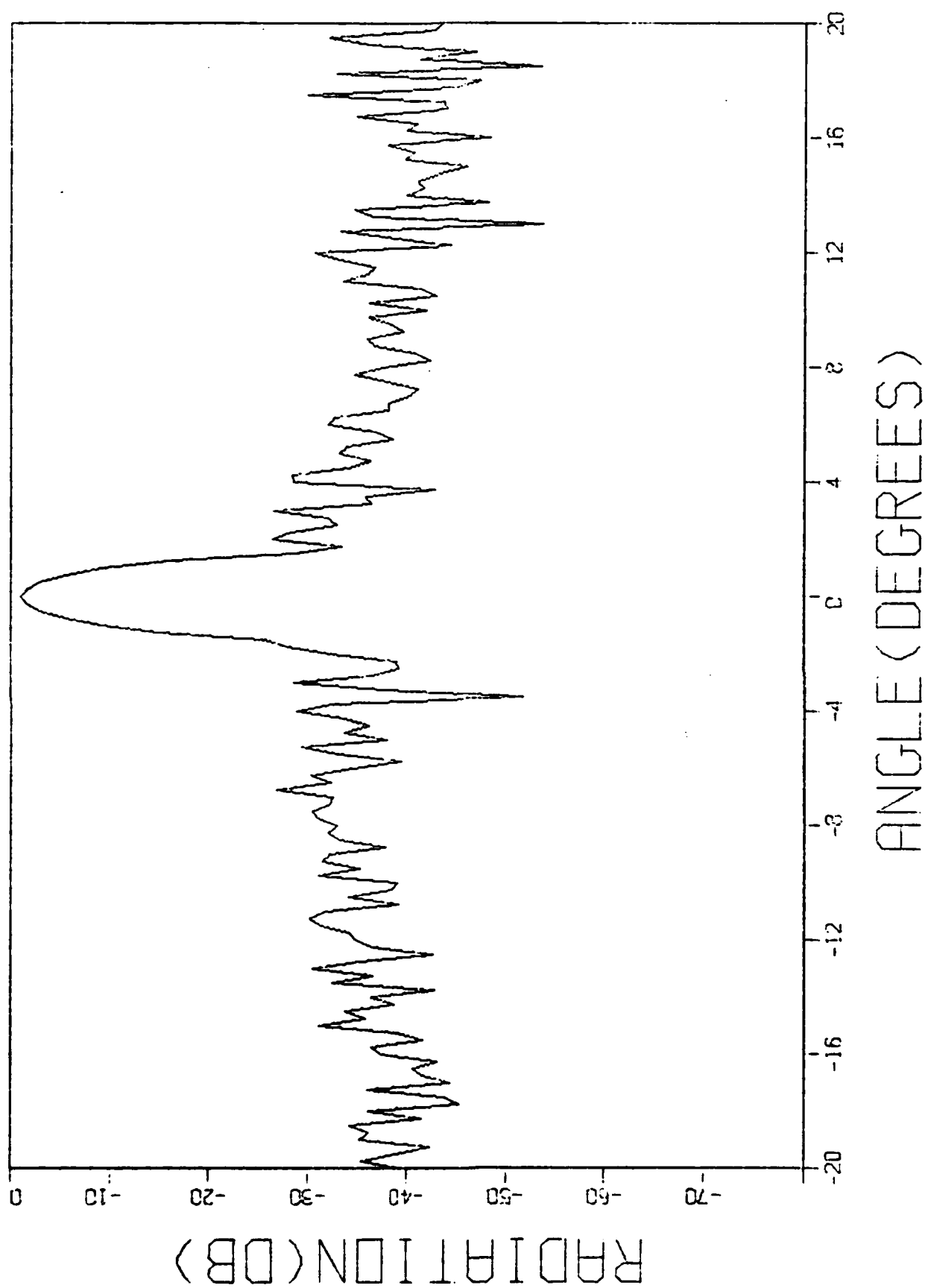


Fig. 4(a) — Radiation pattern, steering angle = 0° , element spacing = $.9\lambda$, 30 dB Chebyshev weighting.

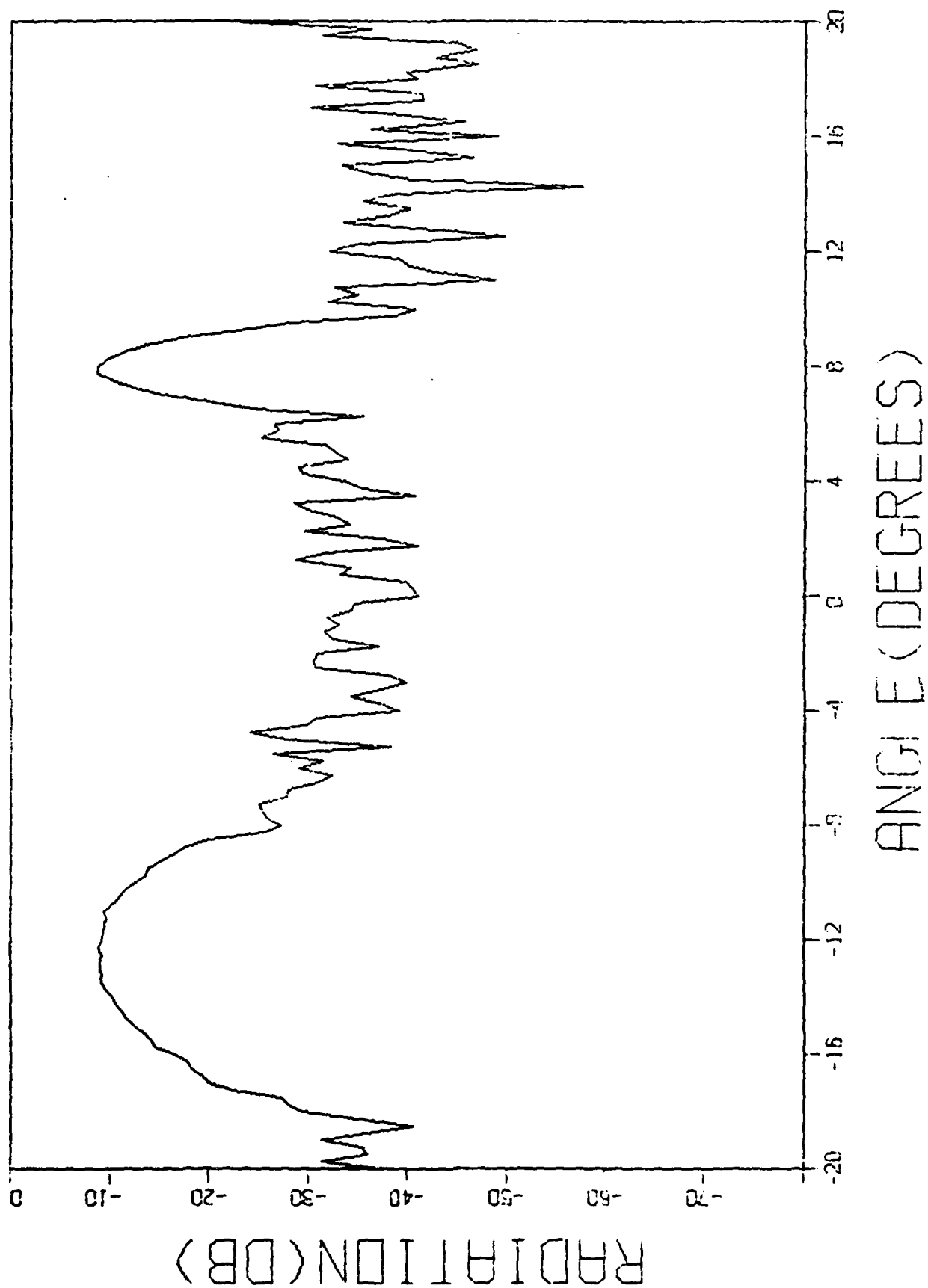


Fig. 4(b) — Radiation pattern, steering angle = -8° , element spacing = $.9\lambda$, 30 dB Chebyshev weighting.

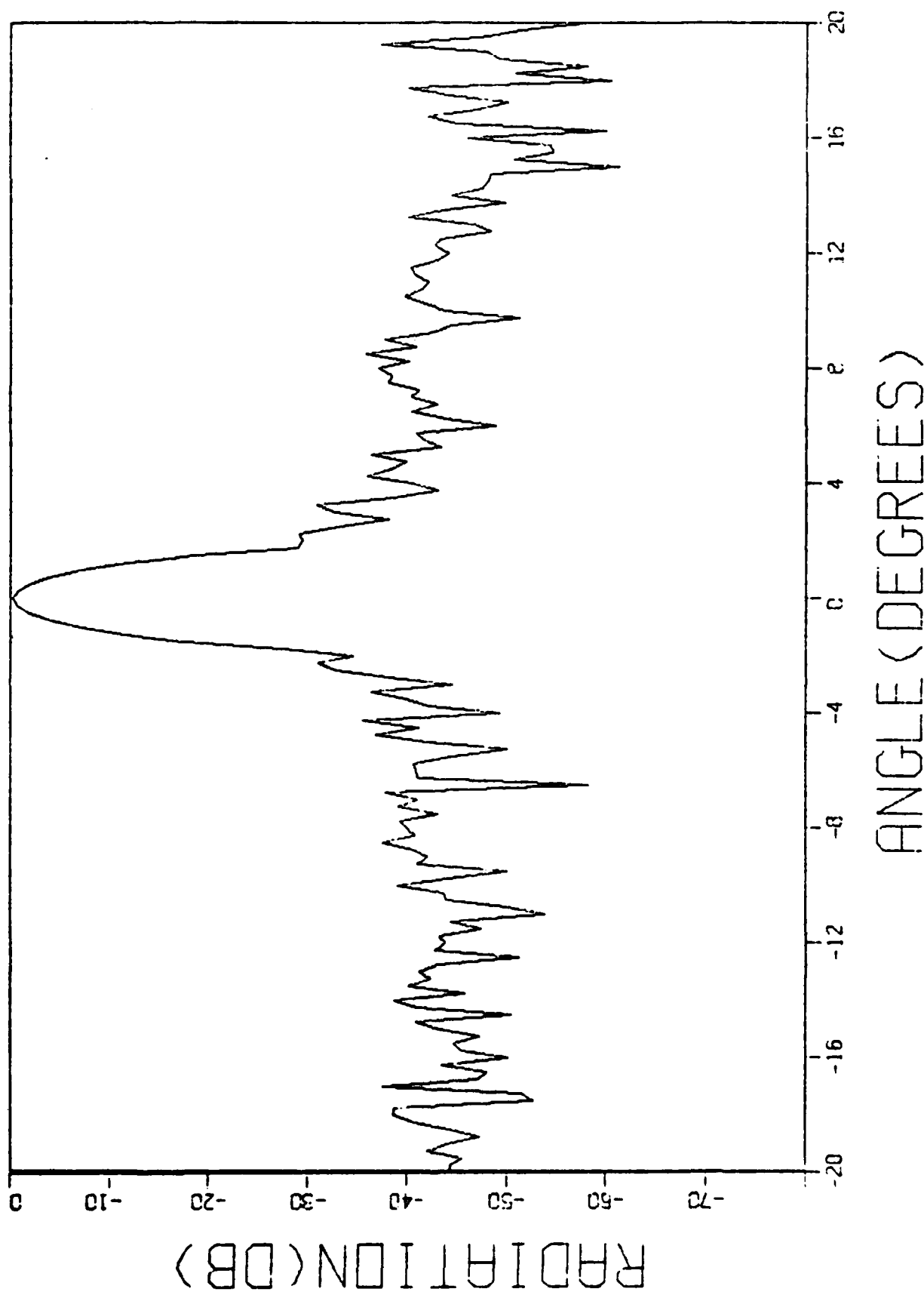


Fig. 5(a) — Radiation pattern, steering angle = 0°, element spacing = $\lambda/2$, 40 dB Chebyshev weighting.

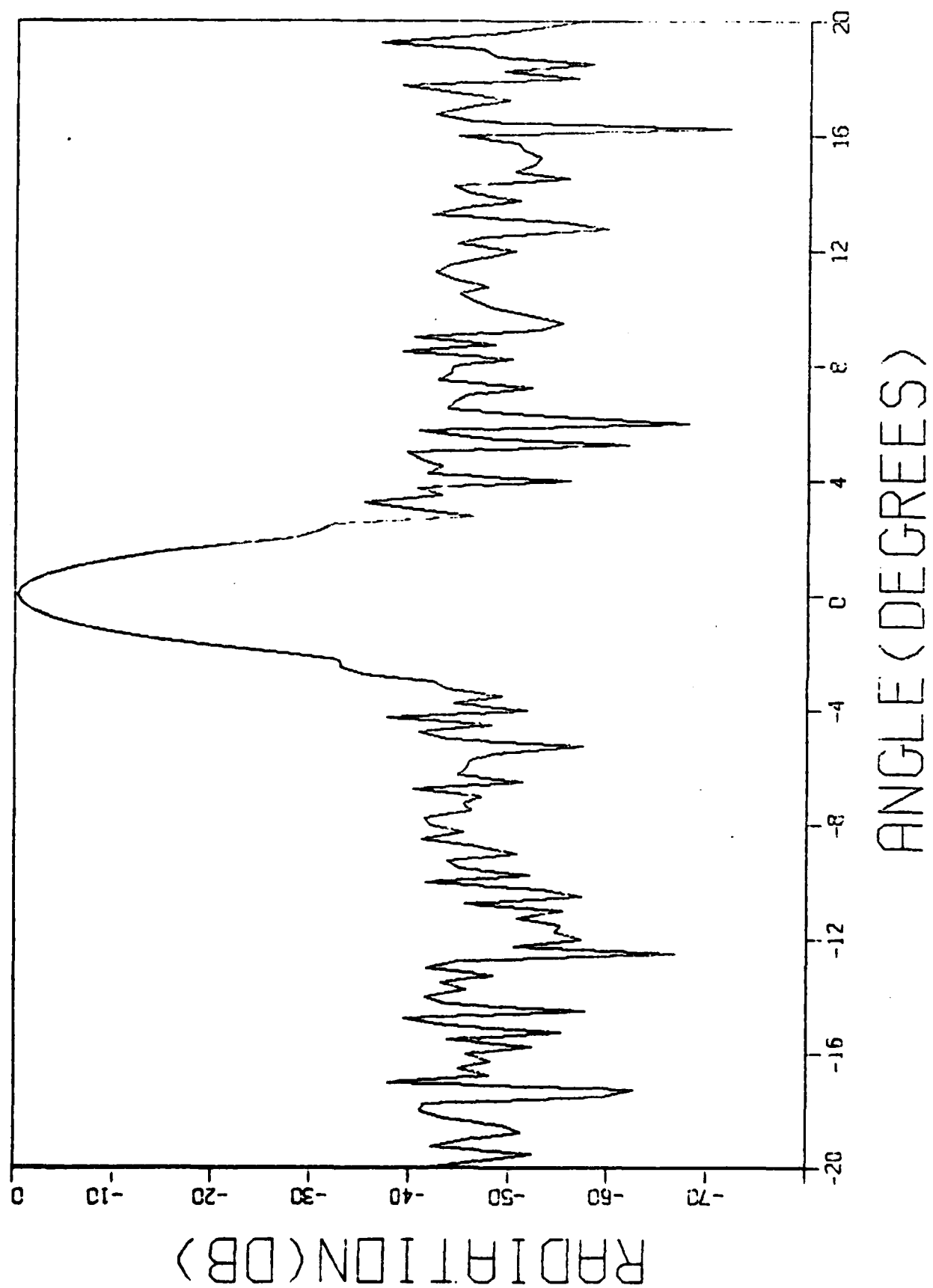


Fig. 5(b) — Radiation pattern, steering angle = 0° , element spacing = $\lambda/2$, 30 dB Chebyshev weighting.

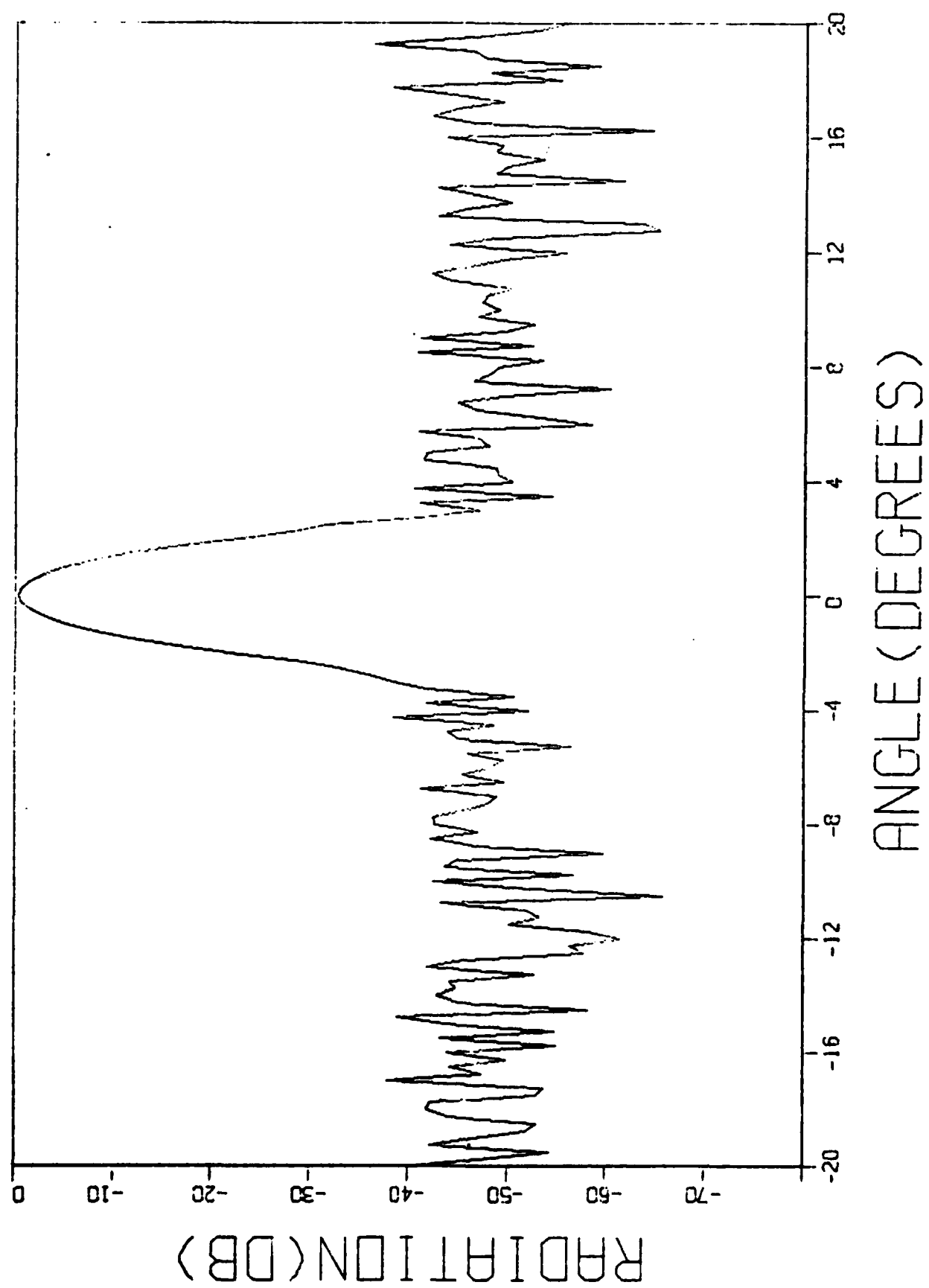


Fig. 5(c) — Radiation pattern, steering angle = 0° , element spacing = $\lambda/2$, 60 dB Chebyshev weighting.

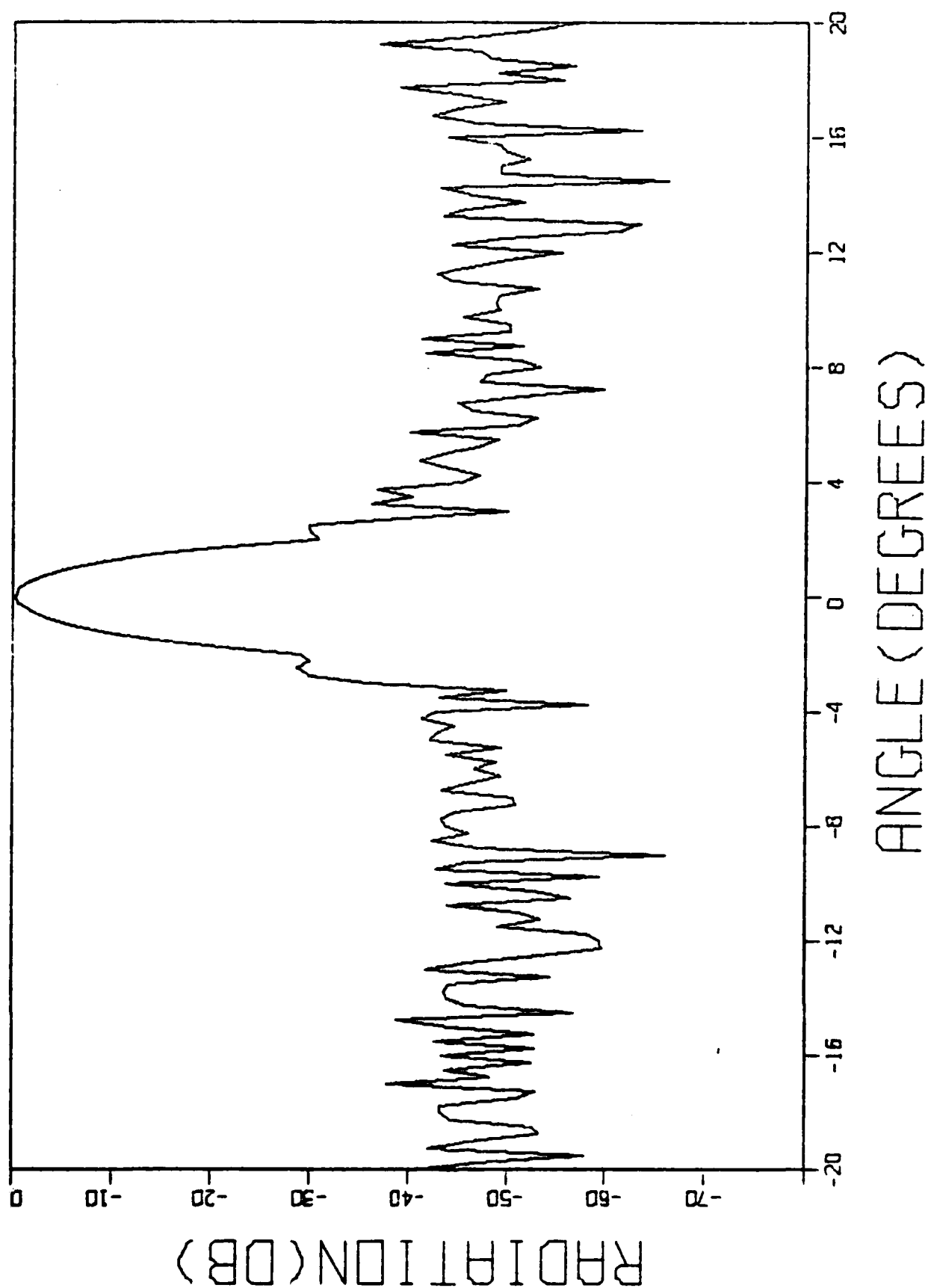


Fig. 6 — Radiation pattern, steering angle = 0° , element spacing = $\lambda/2$, cosine square tapering weight.

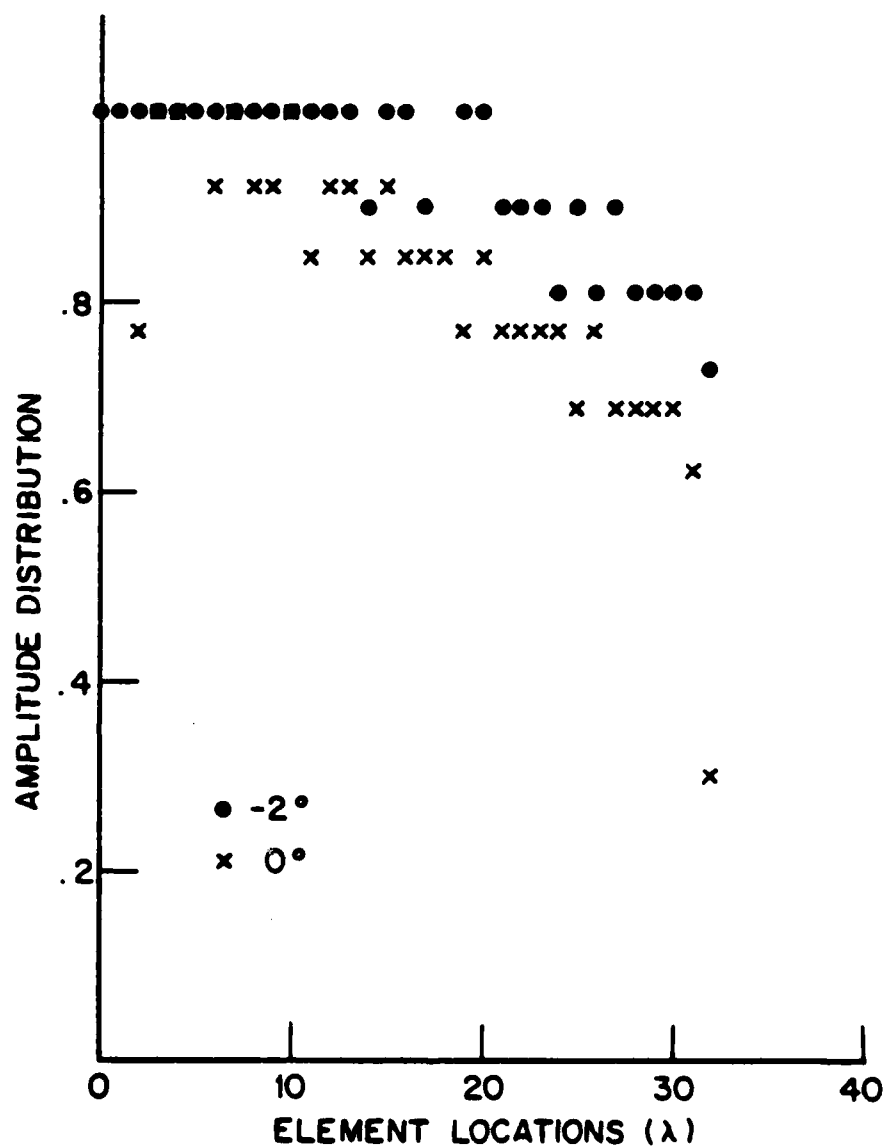


Fig. 7(a) — Array amplitude distribution of plane wave incident at 0° , and -2° with respect to the main reflector.

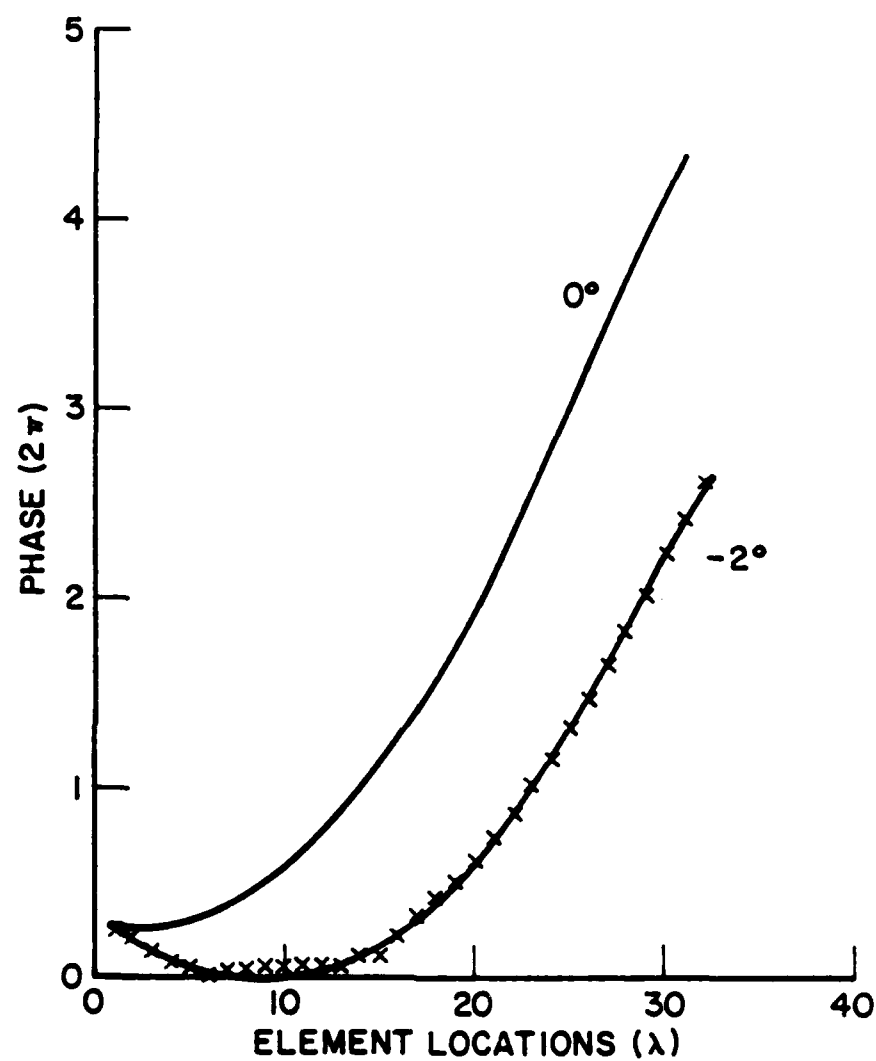


Fig. 7(b) — Array phase distribution of plane waves incident at 0° , and -2° with respect to the main reflector.

APPENDIX A

Figure A1 shows a wave front of a plane propagate towards the main reflector R with an incident angle θ . From this figure,

$$rp = (X_m - X_f) \quad (A1)$$

$$pq = R_o p \tan \theta \quad (A2)$$

$$= (Y_m - Y_f) \tan \theta$$

$$|l_0| = (rp + pq) \cos \theta$$

$$= [X_m - X_f + (Y_m - Y_f) \tan \theta] \cos \theta \quad (A3)$$

where (X_m, Y_m) is a reference point and (X_f, Y_f) is a point on the main reflector. It can be shown that this equation also applies to the case when θ is negative.

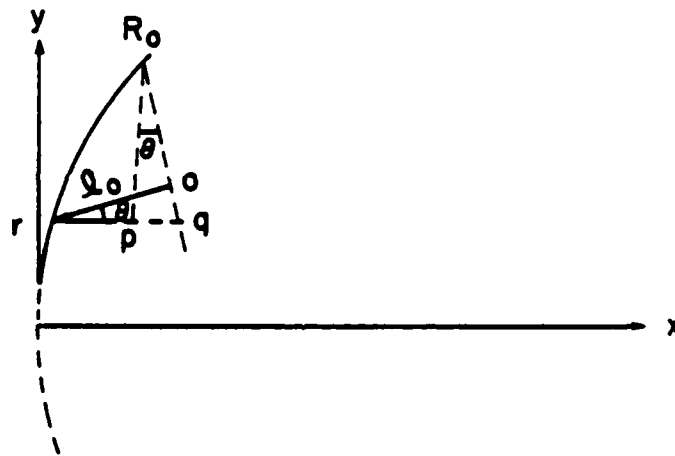


Fig. A-1 — Path length from wave front to main reflector

APPENDIX B

Let a parabola R be described by an equation;

$$y^2 = 4f_m(x+f_m) \quad (B1)$$

where f_m is focal length of the parabola.

The gradient of such a curve is then

$$\nabla G = -2f_m \vec{i} + y\vec{j} \quad (B2)$$

where \vec{i} and \vec{j} are respectively unit vector along x axis and y axis. The unit normal vector \hat{n} is then

$$\hat{n} = \frac{2f_m \vec{i} - y\vec{j}}{\sqrt{y^2 + 4f_m^2}} \quad (B3)$$

An incident wave having incident angle θ with the x-axis, can be represented (See Figure B1).

$$\vec{l}_0 = \cos\theta \vec{i} + \sin\theta \vec{j}.$$

The reflected wave l_1 can be represented

$$\begin{aligned} \vec{l}_1 &= -\vec{l}_0 + 2\hat{n}(\hat{n} \cdot \vec{l}_0) \\ &= -4f_m \frac{x_1 \cos\theta + y_1 \sin\theta}{y_1^2 + 4f_m^2} \vec{i} + 4f_m \frac{x_1 \sin\theta - y_1 \cos\theta}{y_1^2 + 4f_m^2} \vec{j} \end{aligned} \quad (B4)$$

where (x_1, y_1) is the point on the main reflector where the ray l_1 is reflected.

This l_1 can be written into an equation of straight line which has the form

$$y = m_f x + k_f \quad (B5a)$$

$$\text{where } m_f = \frac{y_1 - x_1 \tan\theta}{y_1 \tan\theta + x_1} \quad k_f = \frac{(y_1^2 + x_1^2) \tan\theta}{y_1 \tan\theta + x_1} \quad (B5b)$$

In order to find the length l_1 of the reflected wave propagating from the main reflector to the subreflector, it is required to know the equation which describes the elliptical subreflector. This equation is

$$a_x x^2 + a_y y^2 + (b_x + 2ty)x + b_y y + c_{xy} = 0 \quad (B6)$$

where

$$a_x = (x_e - x_d)^2 - 4a^2 \quad (B7a)$$

$$a_y = (y_e - y_d)^2 - 4a^2 \quad (B7b)$$

$$b_x = (x_e - x_d)a_{xy} + 8a^2x_e \quad (B7b)$$

$$b_y = (y_e - y_d)a_{xy} + 8a^2y_e \quad (B7d)$$

$$t = (x_e - x_d)(y_e - y_d) \quad (B7e)$$

$$a_{xy} = (x_d^2 + y_d^2) - (x_e^2 + y_e^2) - 4a^2 \quad (B7f)$$

where (x_e, y_e) and (x_d, y_d) are respectively the two focal points of the ellipse located at the center of the main reflector and the center of the feed array and a is the distance between these two focal points. Since the ellipse is assumed to pass through the origin, $c_{xy} = 0$.

The intersection point of the reflected ray ℓ_1 and the subreflector is then

$$x_s = B \pm \sqrt{B^2 - C} \quad (B8)$$

The sign is so chosen to yield the larger value of x_s , where

$$2B = - \frac{b_x + 2tk_f + 2a_m k_f + b_y m_f}{a_x + 2tm_f a_y m_f^2} \quad (B9a)$$

$$C = \frac{a_y k_f^2 + b_y k_f}{a_x + 2t_f m_f + a_y m_f^2} \quad (B9b)$$

m_f and k_f are parameters of the reflected ray ℓ_1 which are defined in equation (B5b), and b_x , t , a_y , b_y are defined in equation (B7a) through (B7f).

The y coordinate of this point can be found from (Eq. B5a) when x is known. The length of ℓ_1 is then

$$|\ell_1| = [(x_s - x_1)^2 + (y_s - y_1)^2]^{1/2} \quad (B10)$$

The normal vector \vec{n}' of the ellipse is

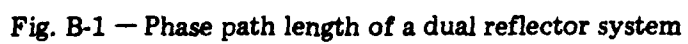
$$\vec{n}' = \frac{n'_1}{\sqrt{n'^2_1 + n'^2_2}} \vec{i} + \frac{n'_2}{\sqrt{n'^2_1 + n'^2_2}} \vec{j} \quad (B11)$$

where

$$n'_1 = 2x_s a_x + 2t y_s + b_x \quad (B12a)$$

$$n'_2 = 2a_y y_s + 2t x_s + b_y \quad (B12b)$$

where (y_s, x_s) is a point on the subreflector.



The reflected ray from the subreflector is then

$$\vec{l}_2 = -\vec{l}_1 + 2\vec{n}'(\vec{n}' \cdot \vec{l}_1) \quad (B13)$$

$$= \frac{a(n_1'^2 - n_2'^2) + 2n_1 n_2 b}{n_1'^2 + n_2'^2} \vec{i} + \frac{b(n_2'^2 - n_1'^2) + 2n_1' n_2' a}{n_1'^2 + n_2'^2} \vec{j} \quad (B14)$$

where a and b are respectively the x and y components of vector \vec{l}_1 (see Eq. (B4)).

The line equation of l_2 is then

$$y = m_a x + k_a \quad (B15)$$

$$\text{where } m_a = \frac{b(n_2'^2 - n_1'^2) + 2n_1' n_2' a}{a(n_1'^2 - n_2'^2) + 2n_1' n_2' b}$$

$$\text{and } k_a = y_s - m_a x_s,$$

where (x_s, y_s) is a point on the subreflector where the ray \vec{l}_2 reflected. If the feed array location is known it is straightforward to find the intersection (x_a, y_a) . The length of \vec{l}_2 is then

$$|\vec{l}_2| = [(x_s - x_a)^2 + (y_s - y_a)^2]^{1/2} \quad (B16)$$

